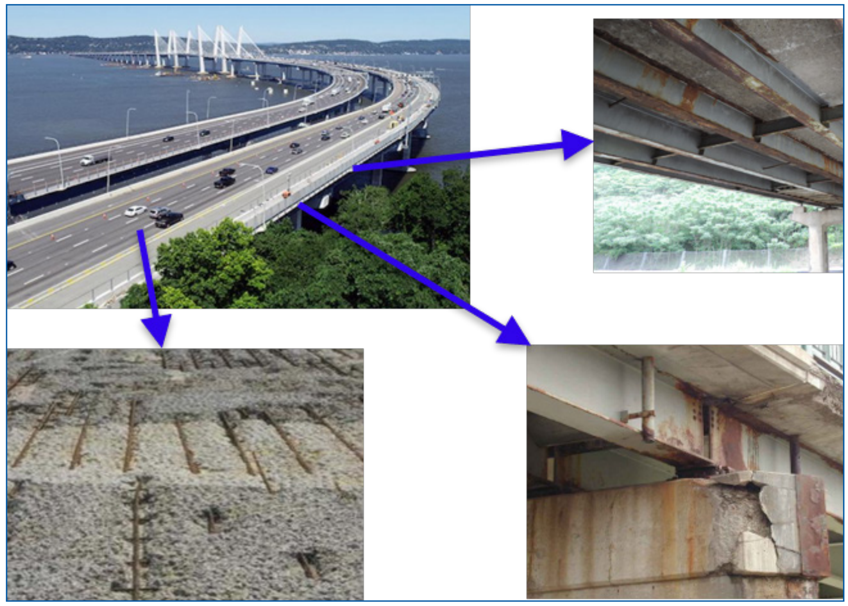
MOUNTAIN-PLAINS CONSORTIUM

MPC 24-560 | G. Jia and M. Li

DEVELOPMENT OF AGE AND STATE DEPENDENT STOCHASTIC MODEL FOR IMPROVED BRIDGE DETERIORATION PREDICTION



MOUNTAIN-PLAINS CONSORTIUM REGION 8

A University Transportation Center sponsored by the U.S. Department of Transportation serving the Mountain-Plains Region. Consortium members:

Colorado State University
North Dakota State University
South Dakota State University
University of Colorado Denver
University of Denver
University of Utah
Utah State University
University of Wyoming

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
MPC-536		

4. Title and Subtitle Development of Age and State Dependent Stochastic Model for Improved Bridge Deterioration Prediction	5. Report Date September 2024 6. Performing Organization Code
7. Author(s) Gaofeng Jia Min Li	8. Performing Organization Report No. MPC 24-560
9. Performing Organization Name and Address Colorado State University Department of Civil and Environmental Engineering Fort Collins, CO 80523	10. Work Unit No. (TRAIS) 11. Contract or Grant No.
12. Sponsoring Agency Name and Address Mountain-Plains Consortium North Dakota State University PO Box 6050, Fargo, ND 58108	13. Type of Report and Period Covered Final Report 14. Sponsoring Agency Code

15. Supplementary Notes

Supported by a grant from the US DOT, University Transportation Centers Program

16. Abstract

Reliable and accurate assessment and prediction of bridge condition deterioration is critical for effective bridge preservation. Deterioration models, combined with current condition information, can guide inspection, maintenance, repair, and rehabilitation planning, and support risk and life-cycle analysis. Existing Markov deterioration models, which assume stationary transition probabilities, often fail to capture the non-homogeneous nature of bridge deterioration influenced by factors such as age, current condition, climate, protective systems, and traffic. This project develops a general age, state, and environment dependent stochastic deterioration model that accounts for these variables. For this purpose, non-homogeneous Markov deterioration models with time-variant transition probabilities are developed. Surrogate models are used to relate these probabilities to explanatory variables such as age, current bridge condition, and operation environmental factors. A Bayesian approach is employed to calibrate the model using bridge inspection and environmental data. By establishing non-homogeneous Markov models, we can better predict bridge conditions. The proposed approach is applied to deterioration modeling for bridges in Colorado. Deterioration models are developed for different types of bridges and different bridge components. Comparisons with existing models showed the advantages of the proposed model.

17. Key Word	18. Distribution Statement
bridges, data mining, deterioration, inspection, maintenance, Markov chains, mathematical prediction, stochastic programming	Public distribution

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	34	n/a

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

DEVELOPMENT OF AGE AND STATE DEPENDENT STOCHASTIC MODEL FOR IMPROVED BRIDGE DETERIORATION PREDICTION

Gaofeng Jia

Min Li

Department of Civil and Environmental Engineering
Colorado State University
Fort Collins, CO 80523

September 2024

Acknowledgement

The funds for this study were provided by the United States Department of Transportation to the Mountain-Plains Consortium (MPC).

Disclaimer

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented. This document is disseminated under the sponsorship of the Department of Transportation, University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.

ABSTRACT

Reliable and accurate assessment and prediction of bridge condition deterioration is critical for effective bridge preservation. Deterioration models, combined with current condition information, can guide inspection, maintenance, repair, and rehabilitation planning, and support risk and life-cycle analysis. Existing Markov deterioration models, which assume stationary transition probabilities, often fail to capture the non-homogeneous nature of bridge deterioration influenced by factors such as age, current condition, climate, protective systems, and traffic. This project develops a general age, state, and environment dependent stochastic deterioration model that accounts for these variables. For this purpose, non-homogeneous Markov deterioration models with time-variant transition probabilities are developed. Surrogate models are used to relate these probabilities to explanatory variables such as age, current bridge condition, and operation environmental factors. A Bayesian approach is employed to calibrate the model using bridge inspection and environmental data. By establishing non-homogeneous Markov models, we can better predict bridge conditions. The proposed approach is applied to deterioration modeling for bridges in Colorado. Deterioration models are developed for different types of bridges and different bridge components. Comparisons with existing models showed the advantages of the proposed model.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

EXECUTIVE SUMMARY

This report presents the development of a non-homogeneous Markov deterioration model for predicting bridge condition transition probabilities, accounting for the influence of age, state, and environmental factors. The primary aim is to improve the accuracy and reliability of bridge condition predictions, which are critical for effective bridge preservation and management.

Methodology

To address the limitations of existing Markov models that assume stationary transition probability matrices, this study employs non-homogeneous Markov models. These models consider the variability in deterioration rates influenced by factors such as age, current condition, climate environment, protective systems, and traffic conditions. A Bayesian approach is used to calibrate the model parameters, integrating bridge inspection and climate data for bridges.

Key Results and Findings

1. **Model Development**: The proposed model uses surrogate modeling to predict time-variant transition probabilities based on explanatory variables. The Bayesian framework allows for the calibration of model parameters using extensive data from the National Bridge Inventory (NBI) and the Long-Term Bridge Performance (LTBP) program.

2. **Application and Results**: The model was applied to concrete and pre-stressed concrete bridges in Colorado. Results indicate that non-homogeneous models provide a better fit for bridge deterioration data, capturing the influence of various factors more effectively, compared with the homogeneous Markov model (which can be treated as a special case of the non-homogeneous Markov model) that only depends on the current condition but neglects the impacts of other factors.

3. **Comparison with Existing Models**: Comparisons with homogeneous Markov models showed that the latter tend to underestimate deterioration rates, especially as bridges age. The proposed non-homogeneous models offer more accurate predictions, which are crucial for timely maintenance and resource allocation.

Implications for Bridge Management

The improved predictive capabilities of the non-homogeneous Markov models have significant implications for bridge management. By providing more accurate deterioration predictions, these models can inform better inspection scheduling, maintenance planning, and budgeting decisions, ultimately enhancing the longevity and safety of bridge infrastructure.

Future Work

Future research will explore incorporating additional factors into the predictive models, conducting sensitivity analyses to identify key determinants of bridge deterioration, and addressing the correlations between the deterioration of different bridge components. Expanding the application of the model to other states and bridge types will further validate its generalizability and robustness.

1. INTRODUCTION

1.1 Background and Literature Review

Reliable and accurate assessment and prediction of bridge condition deterioration is critical for effective bridge preservation. For this purpose, it is important to develop deterioration models that can better predict bridge condition deterioration. Bridge inspection can provide information on current conditions as well as bridge deterioration, which can be used to guide decision making on bridge inspection, maintenance, and preservation.

In order to obtain bridge information in a timely manner, appropriate inspection intervals need to be selected by bridge managers. After the collapse of the Silver Bridge, U.S. bridges are federally required to be inspected every two years based on the National Bridge Inspection Standards (NBIS), and this two-year inspection interval remains the most commonly used interval for bridge inspections (Agrawal et al. 2010; Nasrollahi and Washer 2014). However, such a fixed inspection interval does not consider the effect of the current bridge condition and bridge deterioration on the inspection frequencies for bridges (e.g., bridges in good condition typically require fewer inspections than bridges in poor condition). This could lead to many unnecessary inspections and thus a waste of resources, especially when considering the high cost of bridge inspection activities (Atadero et al. 2019). Recently, there has been a shift toward risk-based inspection planning (Washer et al. 2016a,b; Hesse et al. 2015). For this purpose, stochastic deterioration models for bridge or bridge components are needed.

Different types of stochastic models have been proposed to predict the deterioration and future condition of bridges, e.g., physics/mechanics-based stochastic models (Choe et al. 2008; Kumar et al. 2015; Jia and Gardoni 2018a,b, 2019; Jia et al. 2021), or statistical deterioration models using Markov chains (Agrawal et al. 2010; Wellalage et al. 2014) or Weibull distributions (Agrawal et al. 2010; Nasrollahi and Washer 2014; Li and Jia 2020a). Bridge deterioration is influenced by many factors and is usually a result of many complex (deterioration) processes, e.g., corrosion, concrete degradation, cracking, fatigue (Agrawal et al. 2010). In the absence of good mechanistic-based deterioration models, deterioration models (e.g., Markov chains, Weibull-based models) established based on bridge inspection data are commonly used by state DOTs for bridge asset management and have been incorporated in bridge management systems such as AASHTOWare Bridge (Agrawal et al. 2010). In the Markov chain-based deterioration model, the key is to establish the transition probabilities, which characterize the probability of a bridge or bridge component changing from one condition to another during a specific time interval.

However, existing Markov deterioration models in bridge management systems usually assume stationary transition probability matrix (i.e., assuming homogeneous deterioration process), while in reality the deterioration process is usually non-homogeneous (Devaraj 2009) and the deterioration rate could be different for each bridge or bridge element, considering the differences in influencing factors (or explanatory variables) such as age, climate environment, protective systems, and other external conditions (e.g., traffic conditions). Also, optimization approaches are typically used to establish the transition probability matrix; however, they suffer from drawbacks such as noninvertible matrix and negative transition probabilities (Wellalage et al. 2014). More general stochastic models that can capture the non-homogeneous nature of the deterioration process are needed to accurately predict bridge or bridge element condition deterioration, as are calibration approaches that can establish proper transition probability matrices.

1.2 Proposed Research and Organization of This Report

To address the above challenges, this project develops a general age, state, and environment dependent non-homogeneous Markov deterioration model with time-variant transition probabilities that are modeled as functions of explanatory variables such as age, current bridge condition, and operation environmental factors. A Bayesian approach is developed to calibrate the stochastic deterioration model using bridge inspection data and the environmental information for different bridges. As an example, the proposed approach is applied to establish the deterioration models for bridges in Colorado using bridge inspection and climate data.

The rest of the report is organized as follows. Chapter 2 presents the proposed non-homogeneous Markov models for bridge condition deterioration. Chapter 3 presents the Bayesian framework for calibration of the non-homogeneous Markov model. Chapter 4 presents application of the proposed model to deterioration modeling of bridges in Colorado using bridge inspection and climate data. Comprehensive investigations are carried out to highlight the advantages of the non-homogeneous Markov models. Chapter 5 summarizes the research findings.

2. NON-HOMOGENEOUS MARKOV MODELS FOR BRIDGE CONDITION DETERIORATION

2.1 Markov Deterioration Model

Bridge deterioration can be modeled as a Markov chain, which is a stochastic process where the probability of the future state only depends on the present state (Cesare et al. 1992). This property can be expressed as

$$P(\xi_{t+1} = c_{t+1} | \xi_t\} = c_t, \dots., \xi_1 = c_1, \xi_0 = c_0) \tag{2.1}$$

$$= P(\xi_{t+1} = i_{t+1} | \xi_t\} = c_t$$

where ξ_t represents the random state at the time t with $t \geq 0$, and c_0, c_1, …, c_t are the possible condition states for the bridges or bridge components.

A Markov chain can be completely defined by the initial probability that characterizes the initial state of the chain, and the transition probability that describes the distribution of the next state based on the current state of the chain. Assuming the current state is i and the next state is j, one can denote this transition probability (i.e., given the state i at time t, the probability of the state to be j at time t + 1) is as

$$p_{ij} = P(\xi_{t+1} = j | \xi_t\} = i \tag{2.2}$$

For a bridge or bridge component which has n possible states in total, the transition probabilities can be written as an $n \times n$ transition probability matrix

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \tag{2.3}$$

where $\forall i$ and j, $p_{ij} \geq 0$, and also $\forall i$, $\sum_{j=1}^{n} p_{ij} = 1$. Typically, it is assumed that the condition of bridges or bridge components does not improve (i.e., the "do nothing" assumption) and there is no jumping in condition states (which could happen depending on the inspection interval and the deterioration rate) (Kallen 2009). Therefore, the transition probability matrix $\boldsymbol{P}$ can be simplified to

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & 0 & 0 & 0 \\ 0 & p_{22} & p_{23} & 0 & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.4}$$

$$= \begin{bmatrix} p_{11} & 1 - p_{11} & 0 & 0 & 0 \\ 0 & p_{22} & 1 - p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & 1 - p_{33} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Existing Markov deterioration models in bridge management systems usually assume stationary transition probability matrix (i.e., assuming homogeneous deterioration process). In this case, the transition probability p_{ij} is constant for any time t, which means the deterioration of the bridges or bridge components follows the same pattern whenever it starts. The above transition probability is the so-called one-step transition probability, since it only involves the transition probability from time t to time $t + 1$, e.g., this time step could be a unit inspection interval. Based on the Chapman-Kolmogorov equation, the m-step transition probability matrix is defined as the product of m one-step transition probability matrices (Baik et al. 2006; Devaraj 2009), denoted by $\mathbf{P}^{(m)} = \mathbf{P}^{m}$.

However, in reality, the deterioration process is non-homogeneous (Devaraj 2009), and the deterioration rate could be different for each bridge or bridge element, considering the differences in influencing factors (or explanatory variables) such as age, climate environment, protective systems, and other external conditions (e.g., traffic conditions). For non-homogeneous Markov models, the transition probabilities are functions of the above influencing factors. To establish such functional relationships we use a surrogate model.

2.2 Surrogate Modeling for Transition Probabilities

The idea of a surrogate model to establish a simple mathematical relationship between the inputs and outputs based on a database of results (i.e., input and output pairs). It has been used to establish predictive relationships based on either empirical data or from evaluating the expensive model (e.g., physical experiment or expensive high-fidelity numerical model), with the ultimate goal to maintain the accuracy of the expensive data/model utilized to produce this database while providing greatly enhanced computational efficiency (Sacks et al. 1989; Simpson et al. 1998; Kennedy and O'Hagan 2000; Wang 2003; Rasmussen 2004; Forrester et al. 2008; Jia and Taflanidis 2013; Jia et al. 2016; Li and Jia 2020b). For prediction at new inputs, the surrogate model predictions can be used. Surrogate models can be treated as data-driven models.

For the current problem, we want to establish a surrogate model for the transition probabilities where the input parameters correspond to the explanatory variables such as age, climate environment, and other external conditions like traffic, and the outputs correspond to the transition probabilities. There are many different types of surrogate models, and here we adopt the polynomial regression model to build the relationship between the input parameters and the transition probabilities. Also, to ensure that the prediction of transition probability by the surrogate model is between 0 and 1, instead of using transition probability as output, we select the log-odds of the transition probabilities as outputs, which take value from negative infinity to positive infinity.

The log-odds of the transition probabilities, denoted $y(\mathbf{x})$, can be written as

$$y(\mathbf{x}) = f(\mathbf{x})^T \beta \tag{2.5}$$

where $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ represent the n_x explanatory variables, $\mathbf{f}(\mathbf{x})$ is the q-dimensional vector of basis function, e.g., linear or quadratic functions of $\mathbf{x}$. For the linear and full quadratic cases, q equals to $(n_x + 1)$ and $(n_x + 1)(n_x + 2)/2$, respectively. $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_q]^T$ is the regression coefficients vector.

Once calibrated/established, the surrogate model can be used to predict the log-odds of the transition probabilities for any new input $\mathbf{x}$, and we will denote this prediction $\hat{y}(\mathbf{x})$. Based on the log-odds, in the end, the one-step transition probability p_{ij} for any new input $\mathbf{x}$ can be predicted by

$$\hat{p}_{ij}(x) = \frac{e^{f(x)^T \beta}}{1+e^{f(x)^T \beta}} \tag{2.6}$$

For the selection of input parameters, we consider the different influencing factors, such as age, current bridge condition, bridge design parameters (e.g., structural length), climate environment, and other external conditions (e.g., traffic conditions). In comparison, only the current bridge condition is considered when establishing the homogeneous model for bridge deterioration.

As can be seen, the established transition probability $\hat{p}_{ij}(\boldsymbol{x})$ is a function of time and other influencing factors rather than constant, hence the established model corresponds to a non-homogeneous Markov model. In the end, the transition probability matrix will be different for different bridges and bridge components, rather than the same (which is the case for the homogeneous Markov model). The transition probability matrix for a bridge or bridge component with input characteristics $\mathbf{x}$ can be written as

$$\widehat{\boldsymbol{P}}(\boldsymbol{x}) = \begin{bmatrix} \hat{p}_{11} & 1-\hat{p}_{11} & 0 & 0 & 0 \\ 0 & \hat{p}_{22} & 1-\hat{p}_{22} & 0 & 0 \\ 0 & 0 & \hat{p}_{33} & 1-\hat{p}_{33} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.7}$$

where $\hat{p}_{ij}(\mathbf{x})$ is calculated from Eq. (2.6).

Once the deterioration models are calibrated, they can be used to predict deterioration of individual bridges and further for deterioration and condition prediction of the entire bridge inventory (Li and Jia 2021).

3. BAYESIAN FRAMEWORK FOR CALIBRATION OF NON-HOMOGENEOUS MARKOV DETERIORATION MODEL

3.1 Bayesian Updating of Model Parameters

To calibrate the model parameters (i.e., $\boldsymbol{\beta}$) in Eq. (2.6), Bayesian updating is used. Bayesian model updating can provide the updated probability density of the model parameters (i.e., the posterior) by incorporating information from the dataset and the prior information on the model parameters, and it has been used to update statistical distribution models of the time-in-condition rating for bridges (Li and Jia 2020a). Let D denote the available inspection dataset and $p(\boldsymbol{\beta})$ the prior information/distribution for the model parameters $\boldsymbol{\beta}$. Based on Bayes' Theorem, the posterior distribution for $\boldsymbol{\beta}$ can be written as

$$p(\boldsymbol{\beta}|D) = \frac{p(D|\boldsymbol{\beta})p(\boldsymbol{\beta})}{\int p(D|\boldsymbol{\beta})p(\boldsymbol{\beta})\,d\boldsymbol{\beta}} = \frac{L(D|\boldsymbol{\beta})p(\boldsymbol{\beta})}{\int L(D|\boldsymbol{\beta})p(\boldsymbol{\beta})\,d\boldsymbol{\beta}} \propto L(D|\boldsymbol{\beta})p(\boldsymbol{\beta}) \tag{3.1}$$

where $P(D|\boldsymbol{\beta})$ is the probability of observing the data given the model parameters $\boldsymbol{\beta}$, and it is equal to $L(D|\boldsymbol{\beta})$, which is the likelihood function, and the denominator is a constant known as the "evidence" $P(D) = \int\{P(D|\beta\})p(\beta)d\beta = \int\{L(D|\beta\})p(\beta)d\beta$. The prior is usually selected to reveal our initial knowledge of the model parameters, where either informative or non-informative priors can be used. To establish the posterior distribution, one key part is to establish the likelihood function for the dataset D.

When the number of data increases or is large, the posterior distribution typically concentrates in a small region, and for such cases point estimates of the model parameter, either the maximum a posteriori (MAP) or maximum likelihood estimate (MLE), can be used to establish an estimate of the model parameter.

3.2 Maximum Likelihood Estimation

To calibrate the model parameters (i.e., $\boldsymbol{\beta}$) in Eq. (2.6), MLE is adopted in this project. Assume that the observed condition states of a specific bridge or bridge component are $c_1, \ldots, c_n$, then the likelihood of observing any possible pair of the states (i, j) is equal to the product of the transition probabilities (Kallen and Van Noortwijk 2006)

$$L(c_1, \ldots, \ldots c_n|\beta\} = \prod_{k=1}^{n_{obs}} p_{ij}(x^k) \tag{3.2}$$

where n_{obs} is the number of the observations of the state pairs (i, j) for a specific bridge or bridge component. Since the inspection of each bridge system is independent from others, the likelihood of the condition state pairs for all bridges in the bridge inventory can be calculated by simply multiplying the likelihood for each bridge or its component

$$L(c_1, \ldots, \ldots c_n|\beta\} = \prod_{l=1}^{N}\prod_{k=1}^{n_{obs,l}} p_{ij}(\mathbf{x}^{l,k}) \tag{3.3}$$

where $n_{obs,l}$ is the number of the observations of the state pairs (i, j) for the l^{th} bridge or bridge component, and N is the total number of bridges.

To calibrate the model, we need data on bridge conditions over time and also information on the bridges, external conditions, and climate data, which are discussed next. Using the data, the optimal parameters for $\boldsymbol{\beta}$ can be found by maximizing the likelihood function. Numerically, we can maximize the log of the likelihood function, i.e.,

$$\beta^* = \arg\max_{\beta} \log[L(c_1, \dots, \dots c_n|\beta\}) = \arg\max_{\beta} \sum_{l=1}^{N} \sum_{k=1}^{n_{obs,l}} \log\left[p_{ij}\left(\mathbf{x}^{\boldsymbol{l},\boldsymbol{k}}\right)\right] \tag{3.4}$$

3.3 Bridge Condition Data

Bridges in the U.S. are mandated to be inspected under the guidance of the National Bridge Inspection Standards (NBIS), and the associated data are stored in the National Bridge Inventory (NBI), forming a comprehensive source of bridge inspection information (Ghonima et al. 2018). The NBI database contains bridge condition information and uses condition ratings (CRs) to describe the bridge condition. Typically, the CRs are determined by bridge inspectors based on their subjective evaluation of bridges' physical conditions. In the NBI database, CRs ranging from 0 to 9 are assigned to different bridge components (i.e., superstructure, substructure, and deck), where 0 represents the bridge is in a failed condition and 9 represents the bridge is in an excellent condition. Figure 3.1 shows the full descriptions of different CRs used in the NBI database. More details about the CRs and corresponding descriptions used to describe the general condition of bridges or bridge components can be found in FHWA (1995). Here we also show some examples of CR data in the NBI database (see Table 3.1). The NBI database also provides information about the design parameters and traffic conditions. In this work, we will use NBI database bridge information about the CRs, design parameters, and traffic conditions to calibrate the surrogate models for bridge condition deterioration.

Code	Description
N	NOT APPLICABLE
9	EXCELLENT CONDITION
8	VERY GOOD CONDITION - no problems noted.
7	GOOD CONDITION - some minor problems.
6	SATISFACTORY CONDITION - structural elements show some minor deterioration.
5	FAIR CONDITION - all primary structural elements are sound but may have minor section loss, cracking, spalling or scour.
4	POOR CONDITION - advanced section loss, deterioration, spalling or scour.
3	SERIOUS CONDITION - loss of section, deterioration, spalling or scour have seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present.
2	CRITICAL CONDITION - advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored it may be necessary to close the bridge until corrective action is taken.
1	"IMMINENT" FAILURE CONDITION - major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability. Bridge is closed to traffic but corrective action may put back in light service.
0	FAILED CONDITION - out of service - beyond corrective action.

Figure 3.1 Condition ratings (i.e., code in the figure) in NBI database and corresponding descriptions (FHWA 1995)

Table 3.1 Examples of condition rating data in NBI database (STRUCTURE_NUMBER represents the structure number of bridges; DEC_COND, SUPERSTRUCTURE_COND, and SUBSTRUCTURE_CON represent the condition ratings of bridge decks, superstructures, and substructures, respectively)

STRUCTURE_NUMBER	DECK_COND	SUPERSTRUCTURE_COND	SUBSTRUCTURE_COND
0400-4	8	8	8
0400-5	8	7	8
0400-6	8	8	7
0403-1	8	8	8
0403-4	7	7	7
0401-SV1	7	7	7
0401-SV2	7	7	7
0401-SV3	6	7	7
0405-100	7	7	7
0405-101	7	7	7
0405-102	7	7	7
0405-103	7	7	7
0405-104	7	7	7
0405-105	7	7	7
0405-106	7	7	7
0405-107	7	7	7
0405-234	5	6	6
0405-710	7	7	7
0405-1910	6	7	7
0405-3410	8	8	8
0405-5190	6	6	6
0405-7100	7	7	7
0405-7102	7	7	7
0405-7112	9	8	8
0405-farm	7	7	7
ECA-P-001	8	8	8
ECA-P-002	6	8	8
ECA-BT-001	8	8	8
ECA-BT-002	6	6	8
ECA-BT-003	8	8	8
ECA-BT-004	6	8	7
NKA-SFUURD-001	8	8	8

3.4 Bridge Climate Data

While the NBI database provides comprehensive information about the design parameters, traffic conditions, CRs, and other factors for bridges, it does not include the climate environment of the corresponding bridges, which may also have a large impact on bridge deterioration. One goal of this project is to identify the influence of climate/environmental conditions on bridge deterioration; therefore, climate/environmental data are needed, and such data must be collected from other databases. For this, we will use the climate data for different bridges from the LTBP program, which is a Federal Highway Administration (FHWA) long-term research effort to collect research-quality U.S. bridge data to help the bridge community better understand bridge performance (FHWA 2013). These data provide the annual and monthly number of freeze/thaw cycles and snowfalls for a representative sample of highway bridges on a national basis. Therefore, the climate/environmental data are extracted from LTBP to match the bridges in the NBI database. Figure 3.2 shows three examples (corresponding to bridge 0400-4, 0401-SV1, and 0405-100 in Table 3.1) of the annual number of freeze/thaw cycle and snowfall data in the LTBP program.

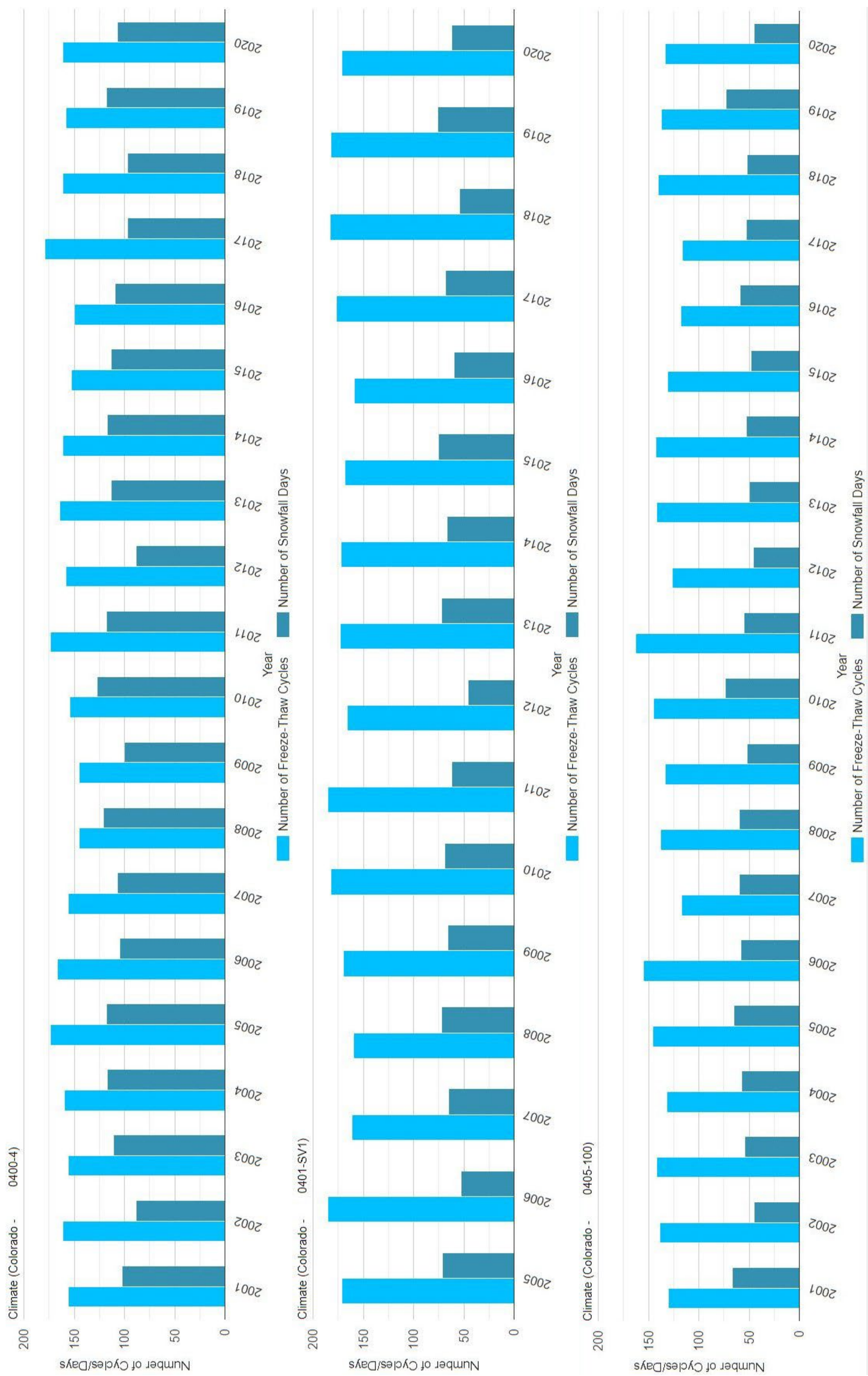


Figure 3.2 Examples of climate data in LTBP program (source: https://infobridge.fhwa.dot.gov/)

4. DETERIORATION MODELING FOR BRIDGES IN COLORADO

To demonstrate the effectiveness of the proposed approach, we apply it to establish non-homogeneous Markov deterioration models for bridges in Colorado. In this example, we consider building deterioration models at the component level. More specifically, for each bridge, Markov deterioration models are established separately for the deck, superstructure, and substructure components. Also, different bridge types are considered in this example, including concrete and pre-stressed bridges. Steel bridges are not considered here since the available data are very limited (see Table 4.1) and might result in low prediction accuracy of the surrogate model.

4.1 Selection of Input Parameters

In this example, non-homogeneous age and state-dependent Markov deterioration models will be established that can explicitly consider the impact of age, condition history, environment, traffic conditions, and other important factors. For this purpose, the model input parameters need to be carefully selected. Note that, to build a good predictive model for the transition probabilities, factors/parameters that have a significant contribution to the bridge deterioration should be included (Li and Burgueño 2010). To determine what parameters significantly impact concrete bridge deck deterioration, one can rely on engineering knowledge, information from existing studies/literature, or statistical analyses to obtain the correlations between the possible parameters and the transition probabilities (Li and Burgueño 2010).

In this example, the model input parameters are selected based on engineering knowledge and the existing research work by Madanat et al. (1995) and Winn et al. (2013). First, current condition rating (CR) and age are expected to impact the decks' future deterioration process. Second, a set of bridge design parameters are selected, i.e., length of maximum span and deck width. Third, traffic condition is also an important factor on bridge deck deterioration, and thus average daily traffic is selected as one of the input parameters. Finally, climate/environmental conditions also contribute to bridge deck deterioration. It is expected that with more freeze/thaw cycles, decks are more vulnerable to deterioration. In the end, the following six variables are selected as input parameters: current CR CR_0, average daily traffic ADT, age Age, deck width $DecWid$, length of maximum span $LenSpan$, and number of freeze/thaw cycles $NumFreeze$.

For comparison purposes, we also built homogeneous Markov deterioration models. The homogeneous Markov model can be treated as a special case of the non-homogeneous Markov model with only the current CR CR_0 as the input parameter (i.e., the transition probability only differs for different CRs without considering impacts of other factors).

4.2 Data Collection and Processing

In this example, 20 years of bridge inspection data from 1998 to 2017 for bridges in Colorado are extracted from the NBI database to develop Markov deterioration models for bridge decks. The following procedures are performed to process the raw NBI CR data: (1) In the database, some CRs may suddenly increase to a higher value (i.e., better condition) in a specific inspection year and then return to the same condition as before in the next inspection year. In such a case, the suddenly changed CR is regarded as noise (or variability in the subjectivity of the inspectors) and replaced by the previous year's CR. (2) Some NBI data show that the CRs improved as the bridge age increased, and the improvement may be the result of bridge reconstruction and repair. Such data are not useful in developing the Markov model for transition probabilities, thus CR data of reconstructed or repaired bridge components are not considered for this analysis. (3) Bridge condition records with missing data are excluded from the analysis.

Note that bridge components sometimes may stay in the same CR after repair or maintenance. For these cases, we currently cannot differentiate them from bridges without repair or maintenance if based only on the inspection data. However, in the future, if we can have bridge inspection information correlated with repair and maintenance information, CRs for bridges with repair or maintenance can also be identified. Also note that bridge decks with CRs equal to 0 to 2 are excluded from the analysis. Such bridges are in seriously poor condition, as described in FHWA (1995), and typically require immediate repair or maintenance.

For the climate data, as mentioned earlier, the number of freeze/thaw cycles is extracted from the LTBP database for all the considered bridges. To match the bridges in the NBI database, a structural number of the bridges is chosen as the criterion considering that the NBI database and LTBP follow the same coding guide.

4.3 Surrogate Model

For the surrogate model, we used linear basis function, and $\mathbf{f}(\mathbf{x})$ corresponds to $[1, x_1, \ldots, x_n]^T$. Note that more generally, higher order basis functions can also be used, e.g., quadratic basis functions, in which case there will be more coefficients to be calibrated. The use of higher order basis function may potentially help the surrogate model to capture more complex variations with the inputs. Here, linear basis function is used to facilitate an easier interpretation of the established surrogate model, and how the outputs are related to each of the inputs x_i. We used the normalized values of the selected input parameters as inputs to the surrogate model. As mentioned earlier, maximum likelihood estimation is used to calibrate the model parameters $\boldsymbol{\beta}$. More specifically, gradient-based optimization with multiple start points is performed to maximize the likelihood in Eq. (3.3) and establish the optimal parameters $\boldsymbol{\beta}$ for the surrogate model. Table 4.1 shows the total number of training data obtained from the NBI database and the LTBP program for developing the surrogate model. As can be observed, we only have limited training data for steel bridges, and the surrogate model trained using the data might have low prediction performance. Therefore, this example only constructs surrogate models for concrete or pre-stressed bridges to predict their deterioration.

Table 4.1 Total number of training data for each type of bridge and each bridge component

	Concrete bridge	Steel bridge	Pre-stressed bridge
Deck	14,411	18	176
Superstructure	15,293	18	206
Substructure	14,795	18	224

4.4 Results for Concrete Bridge Decks

This section shows and discusses the results for the non-homogeneous Markov deterioration model for the concrete bridge decks.

4.4.1 Transition Probability Matrices

The MLE estimates of the coefficients $\boldsymbol{\beta}$ are presented in Table 4.2. Using the established surrogate model, we can predict the time-variant transition probabilities for any bridge deck based on the corresponding $\mathbf{x}$ information.

Table 4.2 MLE estimates of the coefficients β for concrete bridge deck

Basis function	Coefficients	MLE estimates
1	β_1	6.931
CR_0	β_2	-6.220
ADT	β_3	-0.957
Age	β_4	-0.485
DecWid	β_5	0.075
LenSpan	β_7	-0.157
NumFreeze	β_9	-0.092

Table 4.3 shows the transition probability matrices of a specific concrete bridge deck during its service life predicted by the established non-homogeneous Markov model. The other input parameters of the bridge are fixed at: *ADT* = 10000, *DecWid* = 100*ft*, *LenSpan* = 200*ft* and *NumFreeze* = 200. From Table 4.3, we can see that the bridge deck has a smaller probability of staying in the same CR as its age increases. This can be seen by comparing the p_{ii} values for particular CRs for different ages. This means the bridge deck will have a higher probability to deteriorate when it has been in service for many years compared with when it was newly built. Also, looking at the transition probability matrix for a particular age, we can find that when the deck is in relatively good condition (e.g., CR = 9, 8, and 7), it will be more likely to deteriorate in comparison with the case when the deck is in fair or bad conditions (e.g., CR = 6, 5, 4, and 3).

Table 4.3 Transition probability matrices of a concrete bridge deck predicted by non-homogeneous Markov model

Age = 0							
CR	9	8	7	6	5	4	3
9	0.536	0.464	0	0	0	0	0
8	0	0.765	0.235	0	0	0	0
7	0	0	0.902	0.098	0	0	0
6	0	0	0	0.963	0.037	0	0
5	0	0	0	0	0.987	0.013	0
4	0	0	0	0	0	0.995	0.005
3	0	0	0	0	0	0	0.998
Age = 20 years							
CR	9	8	7	6	5	4	3
9	0.514	0.486	0	0	0	0	0
8	0	0.749	0.251	0	0	0	0
7	0	0	0.894	0.106	0	0	0
6	0	0	0	0.960	0.040	0	0
5	0	0	0	0	0.985	0.015	0
4	0	0	0	0	0	0.995	0.005
3	0	0	0	0	0	0	0.998
Age = 50 years							
CR	9	8	7	6	5	4	3
9	0.481	0.519	0	0	0	0	0
8	0	0.723	0.277	0	0	0	0
7	0	0	0.880	0.120	0	0	0
6	0	0	0	0.954	0.046	0	0
5	0	0	0	0	0.983	0.017	0
4	0	0	0	0	0	0.994	0.006
3	0	0	0	0	0	0	0.998
Age = 75 years							
CR	9	8	7	6	5	4	3
9	0.453	0.547	0	0	0	0	0
8	0	0.700	0.300	0	0	0	0
7	0	0	0.868	0.132	0	0	0
6	0	0	0	0.949	0.051	0	0
5	0	0	0	0	0.981	0.019	0
4	0	0	0	0	0	0.993	0.007
3	0	0	0	0	0	0	0.998

For the purpose of comparison, we also use the homogeneous Markov model to calculate the transition probability matrix for the same bridge deck, as shown in Table 4.4. By comparing this table with transition probability matrices for different ages in Table 4.3, we can observe that the homogeneous Markov model may underestimate the deterioration of the bridge deck within an inspection interval, especially when the bridge is initially in good condition and when it is older.

Table 4.4 Transition probability matrix of a concrete bridge deck predicted by homogeneous Markov model

CR	9	8	7	6	5	4	3
9	0.646	0.354	0	0	0	0	0
8	0	0.826	0.174	0	0	0	0
7	0	0	0.925	0.075	0	0	0
6	0	0	0	0.970	0.030	0	0
5	0	0	0	0	0.988	0.012	0
4	0	0	0	0	0	0.995	0.005
3	0	0	0	0	0	0	0.998

4.4.2 Expected Condition Rating Over Time

For the same bridge deck, the variation of the expected CR as the age increases is calculated from the non-homogeneous and homogeneous Markov models. Figure 4.1 shows the expected CR as the age of the bridge deck increases. It can be observed that when the deck age is low (e.g., less than 20 years), the deterioration predicted by both models are relatively close. As its age increases, the difference between the predictions for the expected CR based on the two models becomes larger and larger. Again, this comparison indicates that the homogeneous Markov model is more likely to underestimate the deterioration of the bridge deck considering that the deterioration rate increases as the age of the deck increases. A non-homogeneous Markov model can capture the influence of the age, bridge design parameters, traffic conditions, and climate on the deterioration of the bridge deck, which may more accurately predict the bridge condition deterioration.

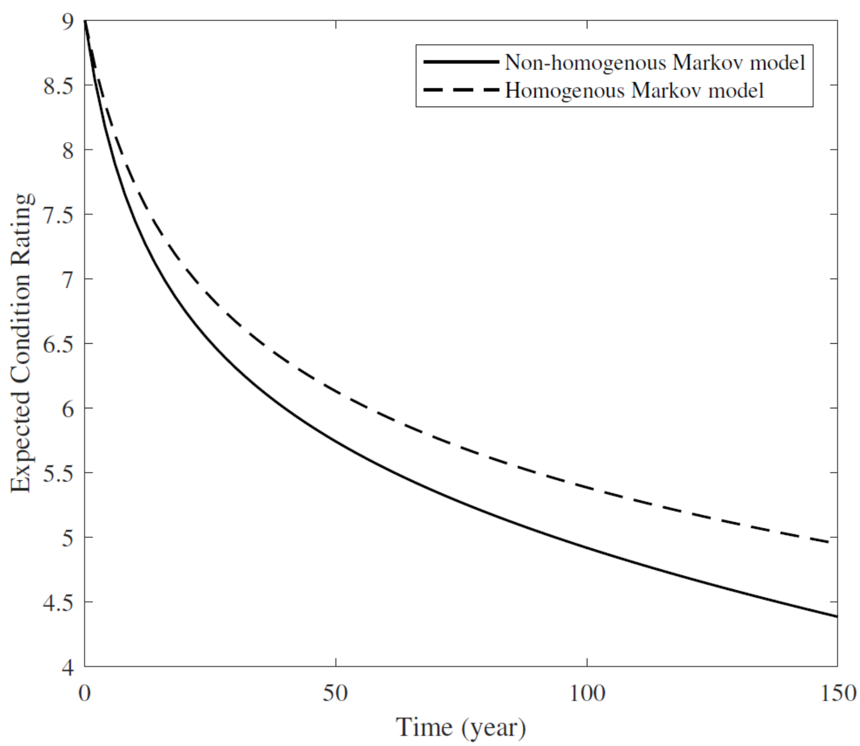


Figure 4.1 Expected condition rating predicted by homogeneous and non-homogeneous Markov model for concrete bridge deck

Next, we applied both the non-homogeneous and homogeneous Markov models to the entire bridge inventory and predicted the deterioration of all bridges 120 years into the future. Such information can be useful for bridge asset management and also guide decision-making regarding bridge inspection, maintenance, and budgeting. Figure 4.2 shows the percentage of the bridge decks in each CR in different years predicted by both models. In this figure, percentage at a time equal to 0 year corresponds to the percentage of the bridge decks in each CR in 2017. Essentially, we are predicting from 2017 how the bridge inventory deteriorates in the future. For both models, the following observations can be made: (1) as time increases, the percentage of bridge decks in CRs equal to 3 and 4 will increase, while the percentage of bridge decks in CRs equal to 7 and 8 will decrease, and such a trend is monotonic for the current bridge inventory; (2) as time increases, the percentage of bridge decks in CRs equal to 5 and 6 first increases and then decreases, and the changing point for CR=6 happens earlier than the one for CR=5.

On the other hand, comparing the predictions by both models, we can find that within a short time period, the percentage of bridge decks that stay in each CR predicted by the homogeneous Markov model is close to that predicted by the non-homogeneous Markov model. However, as time increases, the difference between the predictions by these two models becomes larger. Comparing the values for different CRs, we can see that the homogeneous Markov model overestimates the percentage of bridge decks in CRs 6 and 7 but underestimates the percentage of bridge decks in CRs 3, 4, and 5.

Overall, the results show the importance of including the different influencing variables in bridge deterioration modeling, and the use of non-homogeneous Markov models could provide improved prediction of bridge deterioration. Such information can be used to guide bridge asset management. Also, note that the above observations are for the considered bridge inventory, and the focus is to highlight the differences between predictions by the homogeneous Markov deterioration model and the non-homogeneous Markov deterioration model. If the proposed model is applied to a different bridge inventory, the difference between these two models and resulting trends may vary.

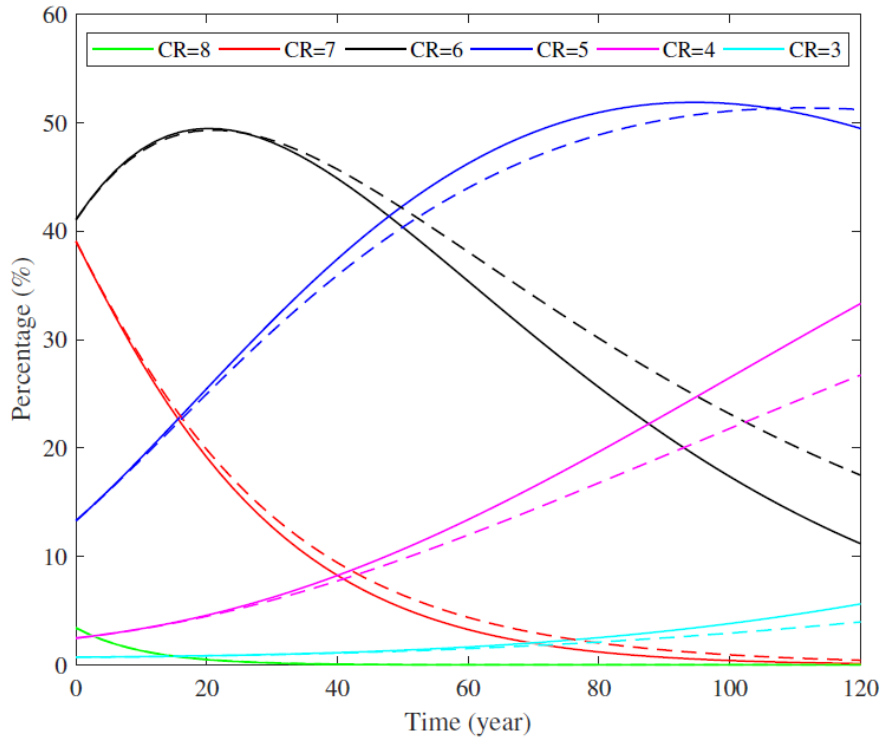


Figure 4.2 Percentage in each condition rating predicted by non-homogeneous (solid lines) and homogeneous (dash lines) Markov deterioration models for concrete bridge deck

4.5 Results of Other Bridges and Bridge Components

In this section, the results from the proposed non-homogeneous Markov deterioration models for other bridges and bridge components are discussed.

4.5.1 Calibrated Model Parameters

The MLE estimates of the coefficients $\boldsymbol{\beta}$ are presented in Table 4.5. From the table we can observe that, for different bridge types, the explanatory variables might have different levels of impact on the deterioration. For example, the current CR CR_0 has a larger impact on the deterioration of pre-stressed bridge components compared with concrete bridge components. Using the established surrogate model, we can predict the time-variant transition probabilities for any concrete or pre-stressed bridge components based on the corresponding $\mathbf{x}$ information.

Table 4.5 MLE estimates of the coefficients β for different bridges and bridge components (C/SUP and C/SUB represent concrete bridge superstructure and substructure, respectively; P/DEC, P/SUP, and P/SUB represent pre-stressed bridge deck, superstructure, and substructure, respectively)

Basis function	Coefficients	MLE estimates				
		C/SUP	C/SUB	P/DEC	P/SUP	P/SUB
1	β_1	6.756	7.027	16.177	11.106	9.482
CR_0	β_2	-5.626	-6.071	-16.557	-10.266	-9.026
ADT	β_3	-0.370	1.608	2.201	2.019	7.526
Age	β_4	-1.307	-2.066	-6.253	-3.948	-2.689
DecWid	β_5	-0.594	-1.046	-1.284	-1.527	-2.154
LenSpan	β_7	0.120	0.936	1.515	-0.180	-0.317
NumFreeze	β_9	0.160	0.654	-1.492	-0.921	-0.791

Similar to concrete bridge decks, we applied both the non-homogeneous and homogeneous Markov models to the entire bridge inventory and predicted the deterioration of all other bridge components over 120 years in the future. Figure 4.3 to Figure 4.7 show the percentage of the concrete bridge superstructures/substructures and pre-stressed bridge decks/superstructures/substructures in each CR in different years predicted by both models. Again, in these figures, percentage at a time equal to 0 year corresponds to the percentage of the bridge decks in each CR in 2017. Note that for pre-stressed bridge components, the best CR observed in 2017 is 7, while for concrete bridge components, the best CR observed in 2017 is 8. From the figures, we first find that for the same type of bridge, the deterioration of different bridge components might show very similar pattern or speed. By comparing the percentages in 2017 for all cases, we then observe that bridge components in very good (e.g., CR=8) or poor (e.g., CR=3 or 4) conditions account for a small proportion, and the bridges with CRs equal to 5, 6, and 7 are more likely to be seen in the inventory.

The proportion of bridges in different CRs experiences significant changes as the time increases. For example, for all cases, the bridge components in CRs equal to 7 account for most of the inventory in 2017; but as 120 years pass, we might not see any bridges staying in CRs equal to 7. This trend holds the same for both the non-homogeneous and homogeneous Markov models. By contrast, the percentage of bridge components in CRs equal to 3 or 4 gradually becomes larger as time increases, and as 120 years pass, bridge components in CRs equal to 3 or 4 will account for a large proportion of the inventory. But note that the increasing trend of pre-stressed bridge components in CRs equal to 4 as predicted by the non-homogeneous Markov model is not monotonic, i.e., increase first and then decrease. Such non-monotonic change can also be seen for concrete and pre-stressed bridge components in CRs equal to 5 or 6. In these cases, we may obtain different trends (i.e., monotonic changes) of the percentages predicted by the homogeneous Markov models, and only the change of the bridges in CRs equal to 6 shows an obvious non-monotonic trend. Overall, during a short time period, the percentages of the bridge components that remain in each CR predicted by both the non-homogeneous and homogeneous Markov models are close, but as time increases, the homogeneous Markov models may underestimate or overestimate the percentage of bridge components in different CRs. Again, based on the results, we can conclude that it is important to include the different influencing variables in bridge deterioration modeling.

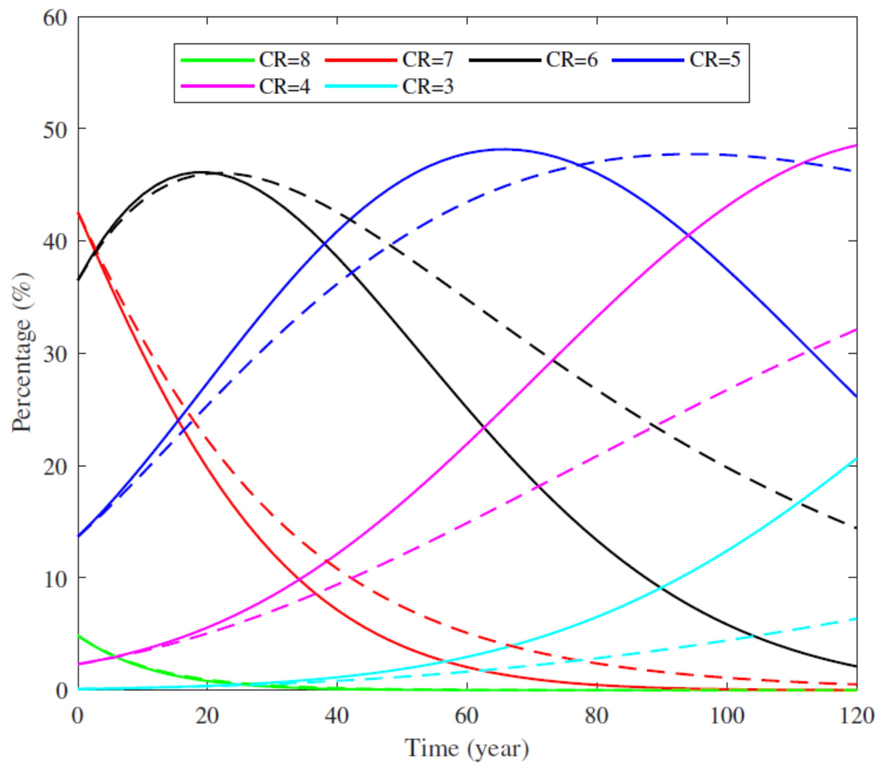


Figure 4.3 Percentage in each condition rating predicted by non-homogeneous (solid lines) and homogeneous (dash lines) Markov deterioration models for concrete bridge superstructure

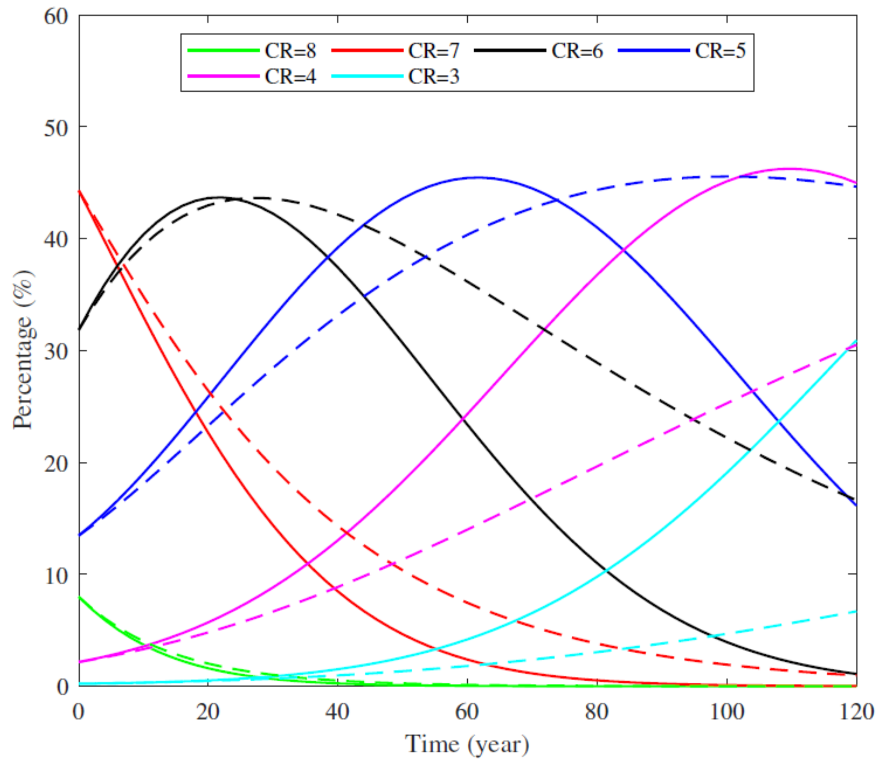


Figure 4.4 Percentage in each condition rating predicted by non-homogeneous (solid lines) and homogeneous (dash lines) Markov deterioration models for concrete bridge substructure

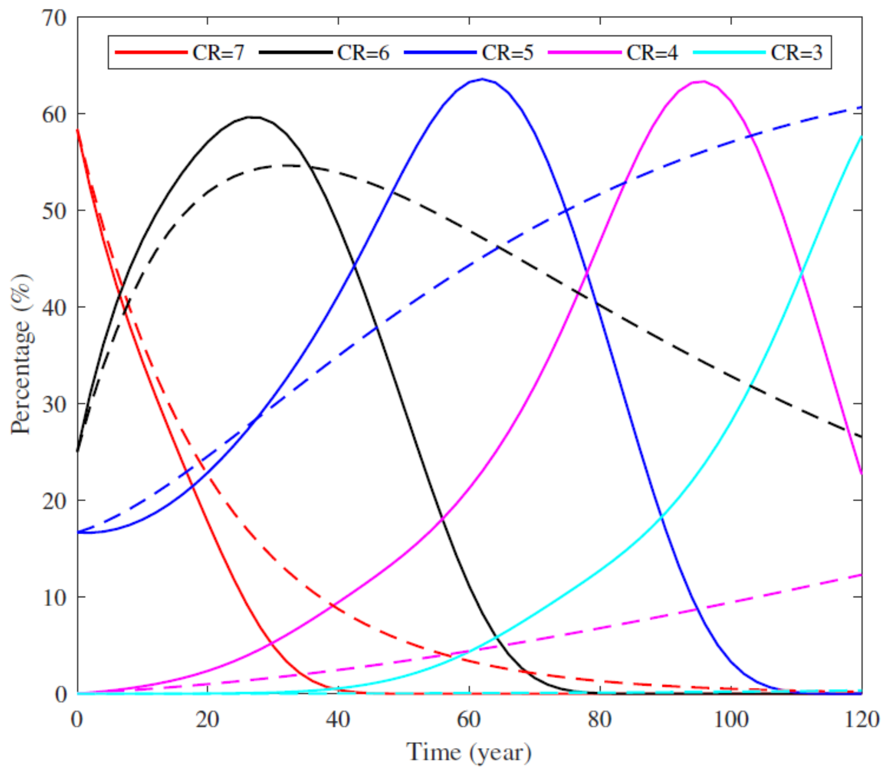


Figure 4.5 Percentage in each condition rating predicted by non-homogeneous (solid lines) and homogeneous (dash lines) Markov deterioration models for pre-stressed bridge deck

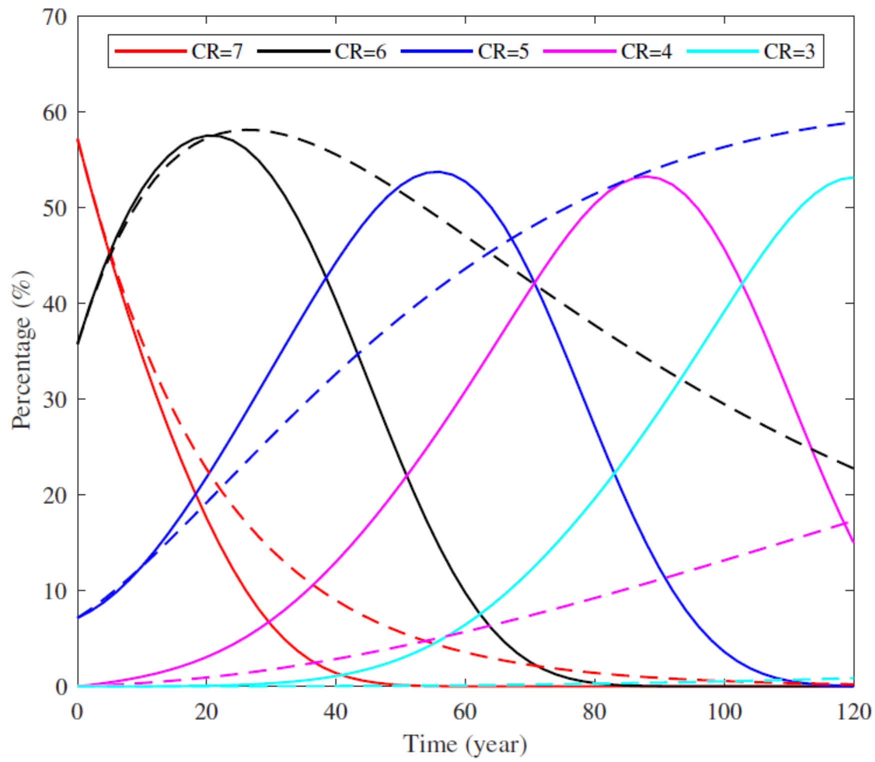


Figure 4.6 Percentage in each condition rating predicted by non-homogeneous (solid lines) and homogeneous (dash lines) Markov deterioration models for pre-stressed bridge superstructure

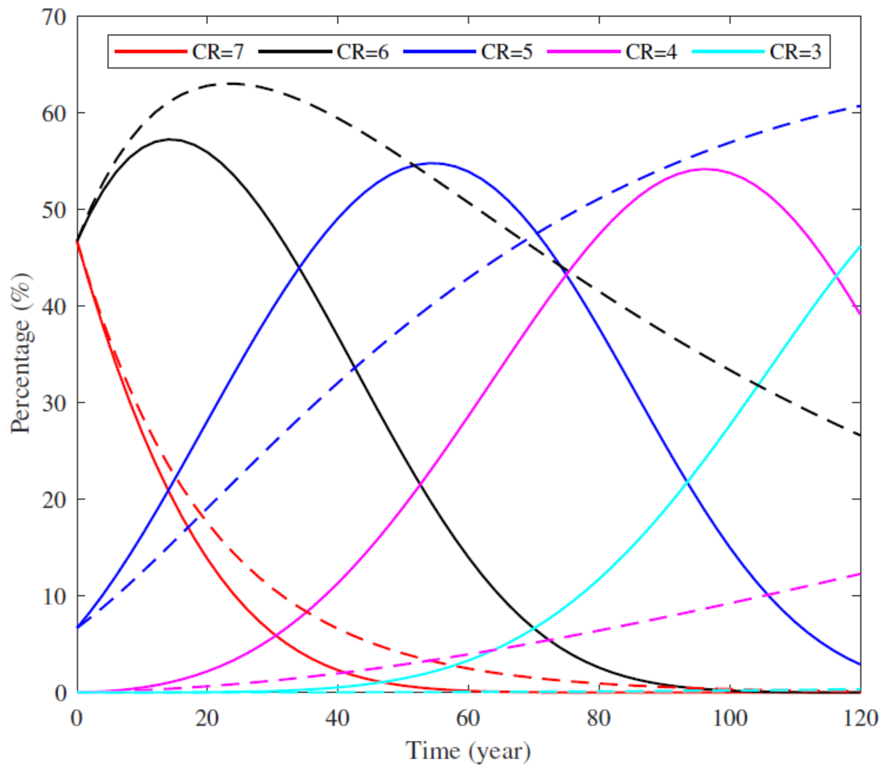


Figure 4.7 Percentage in each condition rating predicted by non-homogeneous (solid lines) and homogeneous (dash lines) Markov deterioration models for pre-stressed bridge substructure

5. CONCLUSIONS

This project developed a non-homogeneous Markov deterioration model for predicting bridge condition transition probabilities. By capturing the impact of various important factors, such as age, state, and environment, the established model allows more accurate and reliable prediction of bridge conditions. As an example, the proposed approach was applied to establish transition probabilities for bridges in Colorado using NBI bridge condition data and LTBP climate data. The results highlighted the importance of including the different influencing variables in bridge deterioration modeling; compared with homogeneous Markov models, the use of non-homogeneous Markov models could provide improved bridge deterioration predictions. Because the current condition is one of the inputs into the model, the proposed model can be used to model deterioration even if the bridge undergoes maintenance actions. In such cases, the updated condition after the maintenance actions will be used as the new/updated input into the model.

The application example mainly focuses on concrete and pre-stressed bridges in Colorado, and different bridge components are considered in the example. However, the proposed non-homogeneous Markov model is general, and it can be easily applied to bridges and bridge components of other states.

Future research will consider including other factors as additional input parameters to inform better predictive deterioration models, especially considering that a variety of information, which has been and continues to be collected, will help the bridge community to better understand bridge performances. In addition, sensitivity analysis on the input parameters may also reveal interesting trends that could be helpful for improving the deterioration models. How to consider the correlations between deterioration of different bridge components when establishing non-homogeneous Markov models for multiple bridge components is also a future research topic of great interest.

6. REFERENCES

Agrawal, A. K., Kawaguchi, A., and Chen, Z. (2010). “Deterioration rates of typical bridge elements in New York.” *Journal of Bridge Engineering*, 15(4):419–429.

Atadero, R. A., Jia, G., Abdallah, A., and Ozbek, M. E. (2019). “An integrated uncertainty-based bridge inspection decision framework with application to concrete bridge decks.” *Infrastructures*, 4(3):50.

Baik, H.-S., Jeong, H. S., and Abraham, D. M. (2006). “Estimating Transition Probabilities in Markov Chain-Based Deterioration Models for Management of Wastewater Systems.” *Journal of Water Resources Planning and Management*, 132(1):15–24.

Cesare, M. A., Santamarina, C., Turkstra, C., and Vanmarcke, E. H. (1992). “Modeling Bridge Deterioration with Markov Chains.” *Journal of Transportation Engineering*, 118(6):820–833.

Choe, D.-E., Gardoni, P., Rosowsky, D., and Haukaas, T. (2008). “Probabilistic capacity models and seismic fragility estimates for RC columns subject to corrosion.” *Reliability Engineering & System Safety*, 93(3):383–393.

Devaraj, D. (2009). “Application of non-homogeneous Markov chains in bridge management systems.” Wayne State University.

FHWA (1995). “Recording and Coding Guide for the Structure Inventory and Appraisal of the Nation’s Bridges.” Technical Report FHWA-PD-96-001, Washington, DC.

FHWA (2013). “Long-term bridge performance (LTBP) program.” Technical report, Washington, DC.

Forrester, A., Keane, A., et al. (2008). *Engineering Design Via Surrogate Modelling: A Practical Guide*. John Wiley & Sons.

Ghonima, O., Schumacher, T., Unnikrishnan, A., and Fleischhacker, A. (2018). “Advancing bridge technology, task 10: Statistical analysis and modeling of us concrete highway bridge deck performance–internal final report.”

Hesse, A. A., Atadero, R. A., and Ozbek, M. E. (2015). “Uncertainty in Common NDE Techniques for Use in Risk-Based Bridge Inspection Planning: Existing Data.” *Journal of Bridge Engineering*, 20(11):04015004.

Jia, G., and Gardoni, P. (2018a). “Simulation-based approach for estimation of stochastic performances of deteriorating engineering systems.” *Probabilistic Engineering Mechanics*, 52:28–39.

Jia, G., and Gardoni, P. (2018b). “State-dependent stochastic models: A general stochastic framework for modeling deteriorating engineering systems considering multiple deterioration processes and their interactions.” *Structural Safety*, 72:99–110.

Jia, G., and Gardoni, P. (2019). “Stochastic life-cycle analysis: renewal-theory life-cycle analysis with state-dependent deterioration stochastic models.” *Structure and Infrastructure Engineering*, 15(8):1001–1014.

Jia, G., Gardoni, P., Trejo, D., and Mazarei, V. (2021). " Stochastic modeling of deterioration and time-variant performance of reinforced concrete structures under joint effects of earthquakes, corrosion, and ASR." *Journal of Structural Engineering*, 147(2):04020314.

Jia, G., and Taflanidis, A. A. (2013). "Kriging metamodeling for approximation of high-dimensional wave and surge responses in real-time storm/hurricane risk assessment." *Computer Methods in Applied Mechanics and Engineering*, 261:24–38.

Jia, G., Taflanidis, A. A., Nadal-Caraballo, N. C., Melby, J. A., Kennedy, A. B., and Smith, J. M. (2016). "Surrogate modeling for peak or time-dependent storm surge prediction over an extended coastal region using an existing database of synthetic storms." *Natural Hazards*, 81(2):909–938.

Kallen, M. (2009). " A comparison of statistical models for visual inspection data." In *Safety, Reliability and Risk of Structures, Infrastructures and Engineering Systems, Proceedings of the Tenth International Conference on Structural Safety and Reliability (ICOSSAR'2009)*, pages 3235–3242. Citeseer.

Kallen, M., and Van Noortwijk, J. (2006). " Statistical inference for Markov deterioration models of bridge conditions in the Netherlands." In *Proceedings of the Third International Conference on Bridge Maintenance, Safety and Management (IABMAS)*, pages 16–19.

Kennedy, M. C. and O'Hagan, A. (2000). " Predicting the output from a complex computer code when fast approximations are available." *Biometrika*, 87(1):1–13.

Kumar, R., Cline, D. B., and Gardoni, P. (2015). "A stochastic framework to model deterioration in engineering systems." *Structural Safety*, 53:36–43.

Li, M., and Jia, G. (2020a). " Bayesian updating of bridge condition deterioration models using complete and incomplete inspection data." *Journal of Bridge Engineering*, 25(3):04020007.

Li, M., and Jia, G. (2020b). "Multifidelity gaussian process model integrating low-and high-fidelity data considering censoring." *Journal of Structural Engineering*, 146(3):04019215.

Li, M., and Jia, G. (2021). " Age, state, and environment dependent non-homogeneous stochastic model for improved bridge deterioration prediction." In *Bridge Maintenance, Safety, Management, Life-Cycle Sustainability and Innovations*, pages 2766–2773. CRC Press.

Li, Z., and Burgueño, R. (2010). " Using soft computing to analyze inspection results for bridge evaluation and management." *Journal of Bridge Engineering*, 15(4):430–438.

Madanat, S., Mishalani, R., and Ibrahim, W. H. W. (1995). " Estimation of infrastructure transition probabilities from condition rating data." *Journal of Infrastructure Systems*, 1(2):120–125.

Nasrollahi, M., and Washer, G. (2014). "Estimating inspection intervals for bridges based on statistical analysis of national bridge inventory data." *Journal of Bridge Engineering*, 20(9):04014104.

Rasmussen, C. E. (2004). " Gaussian processes in machine learning." In *Advanced Lectures on Machine Learning*, pages 63–71. Springer.

Sacks, J., Welch, W. J., Mitchell, T. J., and Wynn, H. P. (1989). "Design and analysis of computer experiments." *Statistical Science*, pages 409–423.

Simpson, T., Mistree, F., Korte, J., and Mauery, T. (1998). "Comparison of response surface and kriging models for multidisciplinary design optimization." In *7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, page 4755.

Wang, G. G. (2003). "Adaptive Response Surface Method Using Inherited Latin Hypercube Design Points." *Journal of Mechanical Design*, 125(2):210–220.

Washer, G., Connor, R., Nasrollahi, M., and Provines, J. (2016a). "New framework for risk-based inspection of highway bridges." *Journal of Bridge Engineering*, 21(4):04015077.

Washer, G., Connor, R., Nasrollahi, M., and Reising, R. (2016b). "Verification of the framework for risk-based bridge inspection." *Journal of Bridge Engineering*, 21(4):04015078.

Wellalage, N. K. W., Zhang, T., and Dwight, R. (2014). "Calibrating Markov Chain–Based Deterioration Models for Predicting Future Conditions of Railway Bridge Elements." *Journal of Bridge Engineering*, 20(2):04014060.

Winn, E. K., Burgueño, R., et al. (2013). "Development and Validation of Deterioration Models for Concrete Bridge Decks—Phase 1: Artificial Intelligence Models and Bridge Management System." Technical report, Michigan Dept. of Transportation.